Online Appendix. Bridging the Gap: Bargaining with Interdependent Values

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0.1 Durable Goods and the Coase Conjecture.

As has been pointed out by Olsen [9] within the context of his linear setting we can interpret the model also as a durable good problem in the following way. Let v be distributed according to $G(v)$ (which can be obtained from $v(c)$ and the distribution $F(c)$) and let $\widetilde{c} \equiv v^{-1}$. Then $1 - G(v)$ is the demand function where the total mass of customers has been normalized to 1. Let $V(Q)$ be the corresponding inverse demand function. Let q_t be the quantity sold at time t, and Q_t be the cumulative sales including time t :

$$\begin{aligned} Q_t &= Q_{t-1} + q_t \\ Q_0 &= q_0 \end{aligned}$$

Assume the following industry-wide experience effects: if the industry sold Q_{t-1} units by time t and sells q_t units at time t then the cost per unit of production at time t is

$$\begin{aligned} C(Q_{t-1}, q_t) &= \int_{Q_{t-1}}^{Q_t} \frac{\widetilde{c}(V(q))}{q_t} dq \text{ if } q_t > 0 \\ C(Q_{t-1}, q_t) &= \widetilde{c}(V(Q_{t-1})) \text{ if } q_t = 0 \end{aligned}$$

For example, if demand is linear, $V(Q) = 1 - Q$, and the underlying function from the bargaining problem is $\widetilde{c}(v) = \frac{v}{\eta}$ for some $\eta > 1$, then

$$C(Q_{t-1}, q_t) = \frac{1}{\eta}\left(1 - Q_{t-1} - \frac{q_t}{2}\right)$$

Our assumption that $v'(c) > 0$ implies that the costs of serving customers fall with the cumulative sales. $C(Q_{t-1}, q_t)$ here corresponds in the bargaining model

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to the average cost of serving the buyer conditional on a sale, completing the usual analogy between the durable goods and bargaining problems.

A competitive equilibrium in this model is described by a sequence of prices $\{p_t\}_{t=0}^{\infty}$ and quantities $\{q_t\}_{t=0}^{\infty}$ such that:

$$\begin{aligned} p_t &= C(Q_{t-1}, q_t) \text{ if } q_t > 0 \\ p_t &\leq C(Q_{t-1}, q_t) \text{ if } q_t = 0 \\ V(Q_t) - p_t &= \delta (V(Q_t) - p_{t+1}) \end{aligned}$$

The first condition is that if there are any sales in period t then firms make zero profit. The second condition means that firms are always willing to supply the product at prices above the current marginal cost, so for sales to be zero it must be that prices are below current marginal costs. The last condition captures buyer optimality: given the sequence of prices buyers choose optimally when to buy.

Note that even if the DL equilibrium has zero profit in the limit $\Delta \to 0$ (which is a non-generic case), it does not satisfy the conditions of competitive equilibrium: during the quiet periods prices are necessarily higher than $C(Q_{t-1}, q_t)$.

Finally, we stress the positive externality in this market: a firm producing more today reduces the marginal cost to other firms both in the current period and in the future. Since a firm does not capture that increase of total surplus, the competitive equilibrium outcome is inefficient: Q_t is growing too slowly. It is efficient to produce immediately at the level where $\widetilde{c}(V(Q)) = V(Q)$, but the competitive equilibrium is only slowly converging to that total output.

In the continuous-time limit prices change continuously over time with $p_t = \widetilde{c}(V(Q_t))$ and cumulative sales follow the following process:

$$\begin{aligned} Q_0 &= 0 \\ r(V(Q_t) - \widetilde{c}(V(Q_t))) &= -\underbrace{\widetilde{c}'(V(Q_t))\dot{Q}_t}_{=-\dot{p}_t} \end{aligned}$$

That implies:[1]

Proposition 1 *(Weak Coase Conjecture) Consider a durable good problem. Taking the double limit of* $\Delta \to 0$ *and* $g \to 0$, *in case with experience effects, the monopolist equilibrium prices converge to the competitive equilibrium outcome.*

To reiterate, the Coase Conjecture is not about efficiency of the outcome (as it is often interpreted) but about the monopolist acting in the limit as a competitive industry would. With no experience effects, the competitive equilibrium is efficient, but here it is not. This is consistent with the original Coase

[1] This claim was informally discussed in Fuchs and Skrzypacz [6].

(1972) claim that the monopolist without commitment would act no different than competitive sellers.[2]

Finally, we call it the "Weak" Coase Conjecture, since we are not able to prove that all stationary equilibria of the monopolist problem converge in the no gap case to the competitive equilibrium outcome. The claim is just about the double-limit. It is an open question whether a "Strong" version of the conjecture is true, i.e. if all stationary equilibria of the monopolist problem converge to the competitive outcome as $\Delta \to 0$.[3] The methods in Fuchs and Skrzypacz [6] can be used to establish that all stationary equilibria with an atomless limit do converge, but we do not know how to prove or disprove the existence of stationary equilibria of the no gap case monopoly problem with atoms of trade in the limit $\Delta \to 0$.

[2] Since in the standard model competitive equilibrium is efficient, the Coase conjecture is sometimes interpreted as a claim that the monopolist with commitment will also achieve efficiency. Yet, Coase (1972) stresses the comparison between the competitive and monopolistic markets.

[3] Even if all stationary equilibria satisfy the Coase conjecture in the no-gap case there can also also be non-stationary equilibira that violate the Coase Conjecture. See Ausubel and Deneckere (1989) for the construction of such equilibria for the constant marginal cost case.